CFD Modelling of Fully Developed Turbulent Flows of Power-Law Fluids

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Abstract

PHOENICS-3.4 general-purpose CFD software is used to compute the fully developed turbulent flows of power-law fluids in smooth circular pipes, concentric annuli and rectangular ducts. The standard high-Reynolds-number k-ε turbulence model and a modified form of the Lam-Bremhorst low-Reynolds-number k-ε turbulence model, are employed. Fully developed solver of PHOENICS-3.4 is applied for specified volumetric flow rates.

Calculated friction factors and the frictional pressure gradients are compared with the Dodge-Metzner empirical correlation, generalized for non-circular ducts, for the following values of the power-law index n and the generalized Reynolds numbers Re_g: n = 0.47 and 0.70; Re_g = 5000, 10000 and 50000. In cases considered with the modified Lam-Bremhorst k-ε model, the agreement between the PHOENICS predictions and the empirical correlation is within ±8%, which is the typical accuracy of empirical correlations for non-Newtonian flows.

Introduction

A non-Newtonian fluid is one whose apparent dynamic viscosity, i.e. shear stress divided by shear rate, is not constant at a given temperature and pressure but is dependent on flow conditions such as flow geometry, shear rate, etc. sometimes even the kinematic history of the fluid element under consideration (time-dependent fluids) [1-3]. Non-Newtonian fluid behavior is encountered in many chemical and process industries [3].

The most common type of time-independent non-Newtonian fluid behavior observed is pseudoplasticity or shear-thinning, characterized by apparent dynamic viscosity, which decreases with increasing shear rate. The simplest and most commonly used representation of shear-thinning behavior is the power-law model.

The apparent dynamic viscosity of the power-law fluid, μ, is given by:

$$\mu = \tau/\gamma = K\gamma^{n-1} \qquad (1)$$

where τ is the shear stress, γ is the shear rate, K is the fluid consistency index, and n is the power-law index (flow behavior index). For a pseudoplastic (shear-thinning) fluid, the power-law index, n, may have any value between 0 and 1. When n = 1, equation (1) reduces to the equation, $\tau=\mu\gamma$, which describes the Newtonian fluid behavior. In a simple case of incompressible fluid flow in a thin layer between two parallel planes, the shear rate, γ, may be expressed as the velocity gradient in the y-direction perpendicular to that of the shear force (x-direction): $\gamma = \gamma_{yx} = -\partial V_x/\partial y$.

The PHOENICS CFD software was applied previously by Malin [1,2] for modeling the fully developed flows of power-law [1], Bingham-plastic [2] and more general Herschel-Bulkley fluids [2] in smooth pipes. The present paper extends the earlier work [1] to deal with the fully developed turbulent flows of power-law fluids not only in smooth circular pipes, but also in concentric annuli and rectangular ducts. The major objective of the paper is to validate the PHOENICS-3.4 software for the above flows, using empirical correlations for the friction factors and the frictional pressure gradients available in the literature on non-Newtonian flows [3].

Mathematical Model

The transport equations governing the steady-state turbulent incompressible flows of non-Newtonian fluids are described in detail in the earlier papers [1,2] and the PHOENICS-3.4 documentation, which is available on the web site of CHAM Ltd. (www.cham.co.uk). In particular, a modified form of the Lam-Bremhorst low-Reynolds-number k-ε turbulence model was proposed by Malin [1-2] for modeling the non-Newtonian flows in smooth circular pipes.

In this paper, the above equations are applied for modeling the turbulent flows of power-law fluids in a smooth circular pipe, a concentric annulus and a rectangular duct. The flows are assumed to be axisymmetric and fully-developed (in the axial flow direction), and the boundary conditions are needed only at the flow axis and the wall boundaries. At the flow axis, a zero-flux condition is employed for all variables, while at the walls k=0, a zero-flux condition is used for ε and the no-slip condition is applied for the fluid velocity. Both the standard high-Reynolds-number k-ε turbulence model and a modified form of the Lam-Bremhorst low-Reynolds-number k-ε turbulence model, proposed by Malin [1-2], are employed in the present work.

In the case of the modified form [1,2] of the Lam-Bremhorst k-ε turbulence model, the eddy viscosity, ν_t, is determined from the following equation:

$$\nu_t = C_\mu f_\mu k^2 / \varepsilon \qquad (2)$$

The damping function, f_μ, includes the Malin's correction, $n^{1/4}$, which improves the accuracy of the CFD predictions for pipe flows of non-Newtonian fluids:

$$f_\mu = [1-\exp(-0.0165 Re_n/n^{1/4})]^2 (1+20.5/Re_t), \quad Re_n = \sqrt{k} y_n/\nu, \quad Re_t = k^2/(\nu\varepsilon) \qquad (3)$$

where y_n is the normal distance to the wall.

The dumping functions, f_1 and f_2, which are present in the transport equation for ε, are determined from:

$$f_1 = 1+(0.05/f_\mu)^3, \quad f_2 = 1+\exp(-Re_t^2) \qquad (4)$$

The coefficients of the modified turbulence model [1,2] have the same values as in the standard Lam-Bremhorst k-ε model:

$$C_\mu = 0.09, \ \sigma_k = 1.0, \ \sigma_\varepsilon = 1.314, \ C_{1\varepsilon} = 1.44, \ C_{2\varepsilon} = 1.92 \qquad (5)$$

Solution Method

The govering equations are solved numerically with the finite-volume solution procedure by iterations. The fully developed (single-slab) solver of PHOENICS-3.4 is applied for specified volumetric flow rates.

The f_μ modification of the Lam-Bremhorst k-ε model has been implemented in the PHOENICS-3.4 by modifying the subroutine GXLRDF in the GXKE.FOR file. The modified version of GXLRDF is based on the listing of GXLRDF given in [1].

A special care is taken of the proper location of near-wall grid nodes. In most cases considered with the modified Lam-Bremhorst k-ε turbulence model, the non-dimensional distances of these nodes from the walls are about 4 to 10. In one-dimensional cases (pipe and annulus), the above non-dimensional distance is defined as $y^+ = \rho w_* y/K$, where $w_* = (\tau_w/\rho)^{1/2}$ is the friction velocity and τ_w is the wall shear stress.

Results and Discussion

PHOENICS simulation results are obtained for the three geometrical configurations: a circular pipe, a concentric annulus and a rectangular duct. The input data used in the simulations are summarized in Table 1. The following values of the power-law index and the generalized Reynolds number are used: n=0.47, 0.69 and 0.70; Re_g= 5000, 10000 and 50000.

The generalized Reynolds number, Re_g, is defined by the following equation:

$$Re_g = \rho w_b D_h/\mu_{eff}\ , \ \mu_{eff} = K(b+a/n)^n(8w_b/D_h)^{n-1},\ w_b=Q/A \qquad (6)$$

where w_b is the bulk velocity in the axial z-direction, Q is the volumetric flow rate, A is the flow cross section area, D_h is the hydraulic diameter (= 4 times flow cross section area divided by wetted perimeter) and μ_{eff} is the effective dynamic viscosity of the power-law fluid.

The hydraulic diameter, D_h, is given by:

$$D_h = D \text{ (pipe)},\ D_h = D_{out} - D_{in} \text{ (annulus)},\ D_h = 2WH/(W+H) \text{ (rectangular duct)} \qquad (7)$$

where D is the circular pipe diameter, D_{out} and D_{in} are the concentric annulus outer and inner diameters respectively, and W and H are the rectangular duct width and height respectively.

For the circular pipe flow, the values of constants a and b in equation (6) are as follows: a=0.25 and b=0.75. For the flows in concentric annuli and rectangular ducts, these values, which depend on the values of D_{in}/D_{out} and H/W respectively, are given in Table 3.3 on page 134 in [3]. For example, in the cases considered in this paper, a=0.4935 and b=0.9946 for $D_{in}/D_{out} = 0.5$ (annulus) and a=0.27 and b=0.76 for H/W=0.4048 (duct).

The PHOENICS software enables to calculate not only the pressure, the velocity components, the turbulent kinetic energy, k, and its rate of dissipation, ε, but also the socalled STRS, which is equal to τ_w/ρ. The predicted values of $\tau_w = \rho*STRS$ could be compared with experimental data on τ_w available in the literature to validate the PHOENICS predictions.

For the fully developed flows considered in the paper, τ_w is related to the friction pressure gradient, dp/dz, and the Fanning friction factor, f:

$$-dp/dz= 4\tau_w /D_h \text{ , } f = 2\tau_w /(\rho w_b^2) \qquad (8)$$

Friction factors and the friction pressure gradients calculated from equations (8), using the values of τ_w predicted by PHOENICS-3.4, are compared with the semi-empirical Dodge-Metzner correlation, generalized for non-circular ducts [3]:

$$f^{-0.5}= 4n^{-0.75} \log(Re_g f^{(2-n)/2}) -0.4n^{-1.2} \qquad (9)$$

In a circular pipe case ($D_h = D$, a=0.25 and b=0.75), the above equation reduces to the original Dodge-Metzner correlation [1-3], which was proposed to calculate the friction factor, f, in the fully-developed turbulent flows of power-law fluids (polymer solutions and particular suspensions) in smooth pipes for $2900 \leq Re_g \leq 36000$ and $0.36 \leq n \leq 1$. For Newtonian fluids (n=1), the original Dodge-Metzner correlation reduces to the well-known Nikuradse equation.

Table 1 shows the accuracy of PHOENICS predictions in more detail for various values of volumetric flow rates, leading to the following values of Re_g: Re_g = 5000, 10000 and 50000. It is seen that in the cases considered with the modified Lam-Bremhorst k-ε model (rows without *), the agreement between the PHOENICS predictions and the empirical correlation is within ±8%, which is the typical accuracy of empirical correlations for non-Newtonian flows [3]. The accuracy of calculations with the standard k-ε model (rows with *) is lower than that of corresponding calculations with with the modified Lam-Bremhorst k-ε model.

Table 1. Friction pressure gradients predicted with the modified Lam-Bremhorst k-ε model and the standard k-ε model (see rows marked with *)

Circular Pipe, Power Law Fluid

Input Parameter Name	Input Parameter Value	Friction Pressure Gradient
1st Flow Rate (Re_g =5000)	2.725168E-04 m^3/sec	2.014E+3 Pa/m (+8%)
2nd Flow Rate (Re_g =10000)	4.639765E-04 m^3/sec	4.304E+3 Pa/m (-2%)
3rd Flow Rate (Re_g =50000)	1.596246E-03 m^3/sec	3.625E+4 Pa/m (+6%)
3rd Flow Rate (Re_g =50000)	1.596246E-03 m^3/sec	3.735E+4 Pa/m (+10%)*
Diameter	0.0157988 m	
Density	1016.0194 kg/m^3	
Flow Behavior Index (n)	0.6974188317	
Consistency Index (K)	0.0302014439 Pa sec^n	

Concentric Annulus, Power Law Fluid

Input Parameter Name	Input Parameter Value	Friction Pressure Gradient
1st Flow Rate (Re_g =5000)	7.284858E-03 m^3/sec	1.456E+4 Pa/m (-4%)
2nd Flow Rate (Re_g =10000)	1.146512E-02 m^3/sec	2.564E+4 Pa/m (-7%)
3rd Flow Rate (Re_g =50000)	3.286246E-02 m^3/sec	1.362E+5 Pa/m (-4%)
3rd Flow Rate (Re_g =50000)	3.286246E-02 m^3/sec	1.767E+5 Pa/m (+24%)*
Outer Diameter	0.0508 m	
Inner Diameter	0.0254 m	
Density	1318.090 kg/m^3	
Flow Behavior Index (n)	0.4716	
Consistency Index (K)	1.366 Pa sec^n	

Rectangular Duct, Power Law Fluid

Input Parameter Name	Input Parameter Value	Friction Pressure Gradient
1st Flow Rate (Re_g =5000)	3.385031E-03 m^3/sec	4.782E+5 Pa/m (+2%)
2nd Flow Rate (Re_g =10000)	5.749661E-03 m^3/sec	1.087E+6 Pa/m (-1%)
3rd Flow Rate (Re_g =50000)	1.967299E-02 m^3/sec	8.940E+6 Pa/m (+6%)*
Width	0.021336 m	
Height	0.008636 m	
Density	1140.00 kg/m^3	
Flow Behavior Index (n)	0.6916265305	
Consistency Index (K)	0.8658791028 Pa sec^n	

Conclusions

The modified version [1,2] of the Lam-Bremhorst k-ε model has been implemented into the PHOENICS-3.4 CFD software and tested against the generalized Dodge-Metzner correlation [3] on the friction factor (frictional pressure gradient) for a circular pipe, a concentric annulus and a rectangular duct. The agreement between the PHOENICS predictions and the empirical correlation is within ±8% for various generalized Reynolds numbers (Re_g = 5000, 10000 and 50000) with different values of the power-law index (n=0.47, 0.69 and 0.70).

PHOENICS-3.4 software is recommended for CFD analyses of industrial turbulent flows of power-law fluids in smooth circular pipes, concentric annuli and rectangular ducts.

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